MATH 147 QUIZ 7 SOLUTIONS

1. Let R be the triangular region with vertices $(0,0)$, $(1,1)$, $(1,2)$ in the xy-plane. Find the linear transformation that takes the triangle R_0 with vertices $(0,0),(0,1),(1,0)$ in the uv-plane to R. Then use the change of variables formula to find the area of R. (5 Points)

   Recall that a linear transform from the uv-plane to the xy-plane is of the form $(x(u,v),y(u,v)) = T(u,v) = (au+bv, cu+dv)$. Using the as a base, we plug in the requested points to come up with a transformation that works. Note that $T(0,0) = (0,0)$ guarantees the transformation is linear and we have no constant term. Then, $T(0,1) = (b,d)$, and we want this vertex to go to $(1,1)$, so $b = 1$ and $d = 1$. Similarly, $T(1,0) = (a,c) = (1,2)$, so our final linear tranformation is $T(u,v) = (u+v, 2u+v)$. Taking the Jacobian gives

   $$J = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = |-1| = 1.$$

   Lastly, we calculate the area with the change of variables formula.

   $$\iint_R dA = \iint_{R_0} 1\ dA = Area(R_0) = 1/2.$$

2. Let R denote the parallelogram in the xy plane with vertices $(0,0),(1,0),(2,1)(1,1)$. Set up but do not calculate the double integral $\iint_R (y-x)\ dA$ using the change of variables theorem. Hint: Using the inverse transform $u = y-x$, $v = y$ will help find the corresponding region in the uv plane. (5 points)

   As before, we already know the form of a linear transformation. In addition, we have the inverse already, and rearranging tells us that the forward transform must be $T(u,v) = (v-u, v)$. Next, to find the region to integrate over, use the inverse transform on each of the points. Let $S(x,y) = (y-x, y)$. Then $S(0,0) = (0,0)$, $S(1,0) = (-1,0)$, $S(2,1) = (-1,1)$, and $S(1,1) = (0,1)$. That is, the shape in the uv plane is a square with vertices $(0,0),(0,1),(-1,0),(-1,1)$. Lastly, we take the Jacobian of the transform, which is

   $$J = \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = |-1| = 1.$$

   Now we set up the integral.

   $$\iint_R y-x\ dA = \int_{-1}^{0}\int_{0}^{1} u\ dv\ du.$$